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THE IMPORTANCE OF THE NODAL POINTS IN LENS TESTING.*

BY

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THE use of a collimator for obtaining a distant effective source to be used in the optical bench examination of lenses is accompanied by so many disadvantages and difficulties that it is usually dispensed with and an illuminated pinhole at some convenient distance employed in its stead. This procedure at once eliminates all the aberrations which would arise from the use of a collimator lens however well corrected it might be. But it also introduces complications in the geometry of the test which, if disregarded, may sometimes lead to quite erroneous interpretations of some of the results. The nature of the problem can best be seen by first developing the general formulæ and afterward studying the effects of convenient approximations, thereby determining under what conditions the errors introduced by the proposed approximations will have a negligible effect on the results.

This paper deals with the interpretation of lens measurements made with finite object distance on an optical bench equipped with a nodal slide turntable provided with means for holding the lens thereon and for adjusting its position relative to the axis of rotation of the nodal slide. It is further assumed that for the purposes of viewing and locating the image with respect to the axis of the nodal slide a micrometer microscope of good power is mounted on a second carriage, movable on the main track of the bench. Both nodal slide support and microscope carriage should

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be provided with transverse micrometer motions as it is necessary to measure lateral displacements as well as axial ones when studying the characteristics of a lens system.

CARDINAL POINT DETERMINATIONS.

While the nodal slide has long been known, its properties for magnifications other than zero and infinity have not until recently been clearly stated. Within the past few years several papers have appeared on this subject.[1] From these the general law of the nodal slide as developed by Professor Anderson may be briefly stated as follows: An axial image will suffer no lateral displacement for a small rotation of the lens system provided the rotation takes place about a point, called the nul point, so located that it

FIG. 1.

divides the distance between the nodal points into parts whose ratio is equal to the lateral magnification of the system. This will be at once apparent from the similar triangles of Fig. 1. From this it is clear that the axis of rotation of the nodal slide will pass through a nodal point of the system only for the cases of parallel incidence or emergence—on the second nodal point in the first case, on the first nodal point in the second. Thus it is seen that the separation of the nodal points must be taken into account in cases where the magnification is not zero or infinity, and that a knowledge of the location of these points with respect to each other must be had before a true reduction of a set of bench readings can be made, and before even the equivalent focal length can be determined. This knowledge of the relative location of the nodal points may be available from previous experience with other lenses of the same construction. Otherwise a determination of

[1] Prof. A. Anderson, *Phil. Mag.*, 33, Jan., 1917, pp. 157–159; Prof. A. Anderson, *Phil. Mag.*, 34, July, 1917, pp. 76–80; Prof. A. Anderson, *Phil. Mag.*, 34, Sept., 1917, pp. 174–177; R. E. Baynes, *Phil. Mag.*, 33, April, 1917, pp. 357–358; J. A. Tompkins, *Phil. Mag.*, 35, Jan., 1918, pp. 21–27.

the apparent separation of these points must be made. This must be done before it is possible to determine whether the error in reducing the readings on the assumption that the lens was pivoting on the second nodal point is negligible or not.

The general formulæ may be easily obtained from a consideration of Fig. 1. The angles are, of course, exaggerated for the purpose of making the drawing clear. It is assumed that the actual rotations are so small that distortion effects are inappreciable, this being the assumption that is always made in this connection. The axis of rotation is at P, the object being at O and the image at O'. When the lateral shift of the image has been eliminated for a small rotation the triangle OPH and $O'PH'$ are similar. On rotating the slide and lens as a whole through an angle of 180° and again eliminating lateral motion for a small rotation of the lens system all that happens is an interchange of the positions of the two nodal points. That is, the geometry of the figure remains unchanged. But in order to go from one setting to the other, it is necessary to move the lens on the slide through a distance s'—the apparent separation of the nodal points—such that

$$s' = s - 2z$$

in which s is the actual separation of the nodal points and z is the difference between the actual and the apparent position of a nodal point. The formulæ at once follow:

$$s = \frac{a + a'}{a - a'} s'$$

$$z = \frac{a's'}{a - a'} = \frac{a's}{a + a'}$$

$$f = \frac{aa'(a + a' - s)}{(a + a')^2} = \frac{aa'(a - a' - s')}{a^2 - a'^2}$$

$$= \frac{aa'}{a + a'} - \frac{aa's'}{a^2 - a'^2} = \frac{aa'}{a + a'}\left(1 - \frac{s'}{a - a'}\right).$$

The distance, z, from the pivot, P, to the second nodal point is the error made in locating the point on the assumption that it was coincident with the pivot. And $2z$ is the error made in determining the separation of the nodal points on the same assumption. It is to be noticed that for converging lenses, which are the only ones considered in this paper, the error is always such as to make

the nodal points appear to be closer together than they really are.

The last form of the focal length formula is especially convenient as it enables one to quickly estimate the percentage error that would be made in a determination of the focal length by assuming that the nodal points were together on the pivot and using the simple approximate formula with a as the object distance and a' as the image distance. For as soon as s', a, and a' are known the second term in the parenthesis gives the fractional error resulting from the approximation. For instance, if it is desired to determine a focal length of about 100.00 mm. to an accuracy of 0.1 mm., then

$$\frac{s'}{a - a'} \text{ must be considerably less than } 0.001$$

If such proves to be the case it is permissible to use the simpler approximate formula,

$$f = \frac{aa'}{a + a'}$$

thereby simplifying the calculations.

With such object distances as are usually used the errors resulting from this approximation—although negligible in the case of short focus lenses whose nodal points happen to be close together—are not to be neglected in the case of medium or long focus lenses having considerable separation of the nodal points. The formula shows that the percentage error increases with increasing focal length and also with increasing separation of the nodal points. But if these points be coincident, then no error is introduced for any focal length as in such a case the assumptions of the approximation are realized for all magnifications. The error in reducing the focal length readings on a long focus lens with nodal points close together may easily be less than that for one of medium focal length having considerable separation of the nodal points. Therefore, we cannot always say that since the focal length is not large the error in reducing the readings will be negligible if carried out on the assumption that the lens was pivoting on the second nodal point.

Since the second term in the focal length formula is negative, it follows that the approximate computation gives a value of the focal length that is too large for converging lenses with nodal

points normal and too small when the nodal points are reversed, or crossed. It is also of interest to note that when $a = a'$ neither the separation of the nodal points nor the focal length can be determined by this method.

An example will show what the magnitude of the error may be in a case which would hardly be considered exceptional. A Ross Telecentric lens of 13-inch nominal focal length was measured with $a = 8600.0$ mm. The value of a' was 343.1 mm. and for s' it was 61.36 mm.

$$f(\text{approx.}) = \frac{aa'}{a + a'} = 329.94 \text{ mm.}$$

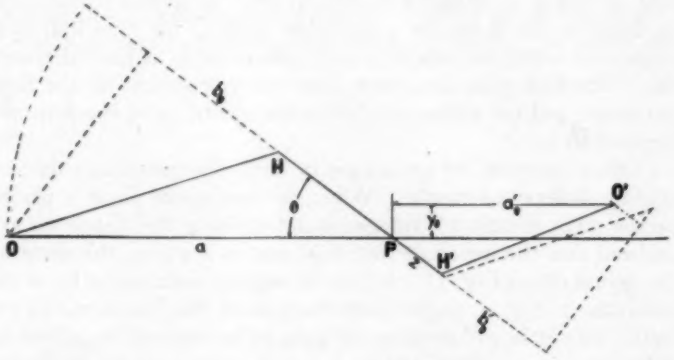
$$f(\text{true}) = 327.48 \text{ mm.}$$

$z = 2.55$ mm., the error in locating H or H' by the approximate method.

FIELD DETERMINATIONS.

To derive the formulæ for the field characteristics assume the lens to be in position for the determination of the cardinal points

FIG. 2.



as just described. The lens may then be rotated about the axis of the nodal slide through any desired half-field angle, θ, as in Fig. 2. Let the image of O be at O'. The Cartesian coördinates, (a'_θ, y_θ), of this point are measured directly with the bench microscope. The normal equation of the theoretical image plane is

$$x \cos\theta + y \sin\theta = z + b'_\theta.$$

It is convenient to express the field aberrations in terms of the length of the perpendicular from O' to the theoretical image plane. This distance is

$$\Delta f = a'_\theta \cos\theta + y_\theta \sin\theta - (z + b'_\theta),$$

the algebraic sign being negative when the image is between the theoretical image plane and the lens. The quantity, z, has already been expressed in terms of the measured quantities in the discussion of cardinal point determinations. To similarly express b'_θ, we have

$$b_\theta = a\cos\theta - (s - z)$$

Therefore,

$$\frac{1}{a\cos\theta - (s-z)} + \frac{1}{b'_\theta} = \frac{1}{f}$$

from which.

$$b'_\theta = \frac{fa\cos\theta - f(s-z)}{a\cos\theta - (s-z) - f}$$

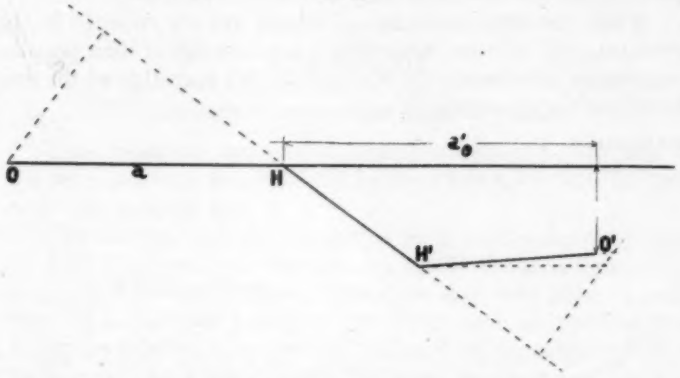
It will be noticed that the actual half-field angle is somewhat larger than the measured angle, θ, when the nodal points are normal, as in Fig. 2, so that a correction must be applied to the angle readings as made on the nodal slide circle if the true half-field angles for which the readings were taken are to be had, although this correction does not enter into the calculation of the field curvature and the astigmatic difference according to the formulæ derived above.

Other methods of procedure in field determinations lead to slightly different formulæ. When the first nodal point is placed on the pivot certain advantages in determining the distortion are realized and the apparent half-field angles are then the same as the actual ones (Fig. 3). When the second nodal point is on the pivot, as in Fig. 4, slight simplification of the field formula results. But it is seldom that the gain to be realized by either of these procedures warrants their use, as it is necessary to determine the actual location of the nodal points before making the field readings in order to properly adjust the lens on the slide.

With these formulæ the reduction of a set of readings leads to reliable values for the field aberrations of the lens under the conditions of the test. What is actually mapped by the lens is not the image of a plane object perpendicular to the axis of the lens but that of a spherical object surface whose centre lies on the

axis of rotation of the nodal slide. Strictly speaking, we cannot deduce from such a set of readings what the field would have been for a plane object at any distance unless we know how the curva-

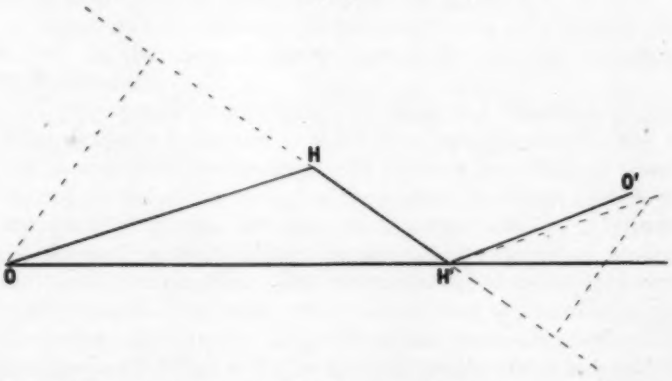
FIG. 3.



ture of field and the astigmatic difference vary with object distance for the lens under consideration. Usually this is not known.

With respect to photographic distortion there is no reduction

FIG. 4.



that rigidly satisfies the problem in general, but an approximation sufficiently good for practical purposes is obtainable in nearly all cases. Erroneous conclusions regarding distortion are most

likely to be met when dealing with systems having considerable separation and displacement of the nodal points with respect to the lens, as in the case of telephoto combinations. But whenever the emergent pencil is symmetrical with respect to the emergent principal ray the distortion may be exactly evaluated.

While the refinements here discussed are not required in the great majority of cases, nevertheless a knowledge of their possible importance is necessary for the satisfactory operation of the lens bench method of testing.

ROCHESTER, N. Y.,
January 14, 1920.

THE MEASUREMENT OF DIFFUSE REFLECTION FACTORS AND A NEW ABSOLUTE REFLECTOMETER.

BY

A. H. TAYLOR.

I. INTRODUCTION—NATURE OF REFLECTION.

THE reflection factor of a surface is defined as the ratio of the total luminous flux reflected by the surface to the total luminous flux incident upon it.

The surface may be illuminated by a narrow beam of light, by light from several directions, by totally diffused light, or by some combination of these. Reflection may take place in many ways, *e.g.,* specular reflection, in which case an incident cone of light is reflected as a cone, the angles of incidence and reflection being equal; perfectly diffused reflection, in which case the light is reflected in all directions in accordance with Lambert's cosine law; mixed specular and diffused reflection, in all possible combinations between the extremes.

No surface obeys Lambert's cosine law perfectly, and most surfaces are very far from being perfect diffusers.

The numerical value of the reflection factor of a surface may depend on the color of the incident light and the manner of its incidence.

In the practical application of light and illumination it is often desirable to be able to determine the reflection factors of various surfaces. In the design of a lighting installation a knowledge of the reflection factors of the walls and ceilings enables the illuminating engineer, with the aid of prepared tables, to estimate the size and number of lamps which will be required to produce the desired illumination. The determination of reflection factors with precision is one of the most difficult feats in photometry, and previous to this time, so far as the author knows, no method has been proposed which will give accurate results and which may be applied to the measurements of surfaces in place. All previous methods, with the exception of one, involve laboratory measurements, and the one exception does not give correct results.

II. EARLIER REFLECTOMETERS.

If one considers the way in which the light flux is reflected, it is evident that in all cases, except that of pure specular reflection, any method of measuring reflection factors must inevitably involve an integration of the reflected flux. This practically limits the method to some application of the integrating sphere.

A search of the scientific literature reveals only a few articles on this subject. The information obtained shows that several methods have been proposed, but apparently nothing has been done as yet to standardize the method. In all except one of the methods of which descriptions were found the test surface is compared in some way with a standard surface, but little is said about the initial standardization of the reference standard surface. Even though the method of comparison may be faultless, the values obtained are merely relative to that of the standard surface, and if it is incorrectly evaluated they will evidently be subject to the same error. A number of the methods which have been used in the past have been seriously in error because of the assignment of an incorrect value to the standard surface used. A few of the methods used will be briefly described.

Apparently the first definite proposal of an instrument for the measurement of diffuse reflection factors was made by Doctor Nutting [1] in 1912. His instrument consists of a highly polished nickeled ring 150 mm. in diameter and 32 mm. high, through which projects the nose of a Martens-Koenig polarization photometer. An illuminated diffusing glass plate is placed on one side of the ring and the test surface on the other, and the photometer is arranged to view the two surfaces at an angle of about 75° from the normal. He describes it thus: "The principle of the method is that of two parallel infinite planes, one of which is a diffuse illuminator and the other the surface whose reflecting power is to be determined. The relative brightness of the two planes is then the reflecting power of the non-luminous plane." Judging by the low value assigned to magnesium carbonate by this instrument, it apparently gives results which are very seriously in error. Some of the factors which may contribute to this error are as follows:

(*a*) The instrument is based on the principle of parallel infinite planes, whereas it employs planes of very limited area, bounded by a nickeled ring which is far from a perfect reflector.

(*b*) The distribution of illumination over the two planes is seriously disturbed by the presence of the nosepiece of the polarization photometer, and does not have the ideal distribution which is assumed.

(*c*) No surface obeys Lambert's cosine law perfectly, and the brightness of the surface at a very oblique angle is usually very appreciably lower than that of a perfect diffuser emitting the same total flux.

A method devised and used by Mr. W. F. Little [2] at the Electrical Testing Laboratories consists in the projection of a beam of light through a small hole in the wall of an integrating sphere onto the test surface, placed near the centre of the sphere. In this method the brightness of the observation window when the test surface is in place, to that when the standard surface is used, is substantially the same as the ratio of the reflection factors of the two surfaces. Evidently this method is limited by the accuracy of the value assigned to the standard surface, but a slight modification would make it an absolute method. If the light beam is first projected onto the sphere wall at a point unscreened from the observation window and is next projected onto the test surface, screened from the window, the ratio of the brightness of the window in the second case to that in the first case is the numerical value of the reflection factor of the test surface. In this method the area of the test surface should be small with respect to the sphere surface. Another method of using the sphere would be to determine the reflection factor of its surface by a method which will presently be described, then to determine the relative values of test and sphere surface by projecting the beam first on one then on the other, the illuminated spot being screened from the observation window in each case. The greatest practical difficulty in the application of any one of these three methods of using the sphere is the realization of a narrow beam of light which is of a sufficiently high intensity and at the same time is so concentrated that none of it is incident anywhere except on the test surface. The two modifications of Mr. Little's methods, as pointed out by the author, require the sphere surface to be uniform in reflecting power.

In 1916 Doctor Rosa [3] and the present author described and applied a method of measuring the reflection factor of the surface of an integrating sphere. It consists in the determination of

the ratio of the average illumination received by the sphere surface from the test lamp, to the total illumination of the sphere surface by both direct and reflected light, the numerical value of this ratio being the absorption factor of the sphere surface. The absorption factor of an 88″ sphere at the Bureau of Standards, surfaced with a white cement, when the surface was fresh, was found to be 7.5 per cent. Since magnesium oxide and carbonate have long been considered, and probably are, the whitest substances in existence, this test set the lowest possible limit for their reflection factors, and definitely established the fact that the value of 88 per cent. given by Doctor Nutting's reflectometer was considerably in error. This sphere method of determining the reflection factor of the sphere surface is by far the most precise method which it is possible to devise, but it is evidently very limited in its application.

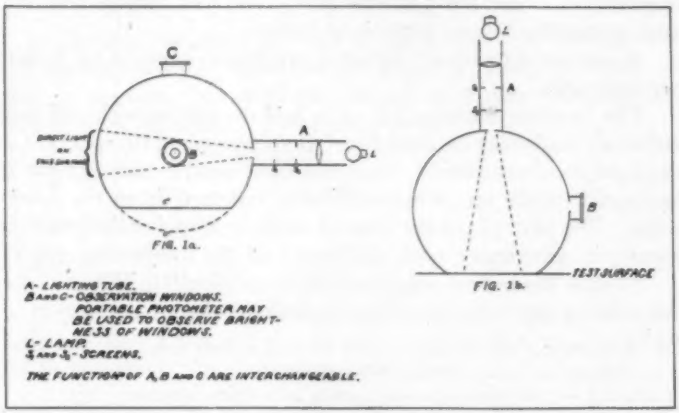
In 1917 Mr. M. Luckiesh [4] described a new relative method for measuring reflection factors. An opal glass globe, such as is used in lighting fixtures, is mounted in a white box. The globe is surrounded by four or more lamps, symmetrically placed. The globe has an opening at the bottom, against which is placed the object to be tested. A brightness photometer views the test object, its brightness being compared with that of a standard surface of known reflection factor. The results may be in error because of the fact that the photometer views the test object at a fixed angle, and that the brightness at that angle may depend very largely on the amount of specular reflection of the object, but it is probable that this error is not very large. This, like other relative methods, will be in error if the standard surface is incorrectly evaluated.

III. THE NEW ABSOLUTE REFLECTOMETER.

In 1916 the author worked out the complete theory of a reflectometer which was to be an absolute instrument, and shortly thereafter the experimental instrument used in these tests was constructed. The few tests which were made gave good results, but for lack of time very little more work was done with the instrument until within the past few months. No publication of the theory was made because insufficient work had been done to completely verify it, but it has now been verified by extensive experiments, as will be shown later.

At the convention of the Illuminating Engineering Society in Chicago in October, 1919, in the discussion of two of the papers presented, the author called attention to the fact that the value of 88 per cent. for magnesium carbonate was much too low, and also briefly described the instrument which it is the object of this paper to describe more fully. Because of delay in publication, that discussion has not appeared in print at the time of writing. In a revision of his discussion, prepared shortly after the convention, the author stated that the reflection factor of magnesium

FIG. 1.



carbonate was approximately 96 per cent., that being slightly lower than a value which he obtained by one measurement by the point-by-point method.*

The reflectometer consists of a small sphere arranged as shown in Fig. 1. At *B* is a small hole through which the opposite wall of the sphere can be viewed by a brightness photometer.

* The absolute method for the determination of the reflection factor of magnesium carbonate which was described by another author in a recent publication is merely a modification of the method which he heard the present author describe in Chicago, and is strictly limited to the determination of the reflection factor of any surface which can be embodied in a hollow sphere. His method uses an incomplete sphere divisible into two fractional parts, whereas the author described the use of the complete sphere and one fractional part.

The segment of sphere surface c' is cut off by a plane, leaving the opening c''. The test surface is placed over this opening, and the direct light is projected onto the sphere wall or the test surface, depending on whether the lighting tube is placed at A or C. The attachments are so constructed that their positions are interchangeable. In the experimental instrument constructed and used in the tests the area c'—the portion cut off— was 10 per cent. of the total sphere area.

IV. THEORY OF REFLECTOMETER.

If a plane surface is illuminated by an infinite plane of uniform brightness, b (candles per unit area), the flux received by unit area of the illuminated plane is πb lumens.[5]

A perfect diffuser of uniform brightness b emits πb lumens per unit area.

The interior illumination of a hollow sphere with diffusely reflecting walls may be considered as composed of two parts: (a) the light received directly from the light source, and (b) the illumination made up of light diffusely reflected from the sphere walls. The part (b) is the same in value at all points in an empty sphere, in accordance with the theory of the integrating sphere.

Assume the hollow sphere arranged as shown in Fig. 1 a. Let the area of the complete sphere be unity.

Let c' = ratio of sphere area cut off to total sphere area.
c'' = ratio of area of hole to total sphere area.
$a = 1 - c'$ = fraction of total sphere area which remains.
m = diffuse reflection factor of sphere surface.
m_x = diffuse reflection facor of test surface.
F = total light flux (lumens) received from lamp.
b_o = average brightness (candles per unit area) of sphere wall, due to *reflected light only* when hole is uncovered.
b_x = brightness when hole is covered with a flat test surface having a diffuse reflection factor m_x.
b = brightness when test surface has reflection factor m.

When the hole is uncovered the escaping light flux may be considered as composed of two parts. The portion of first reflected flux which escapes through the hole is $c'mF$, since it is the once reflected flux which would ordinarily be received by the portion of the surface which has been removed. The second part is the flux due to the average brightness, b_o, of the sphere walls, which in turn is due to reflected light *only*. Its effect in the plane

c'' is equivalent to that of an infinite luminous plane of brightness b_o. Hence this second part is $\pi b_o c''$, and the total flux escaping through the hole is

(1) $F = c' mF + \pi b_o c''$

The total flux received by the remaining portion of the sphere surface is

$$\frac{\pi b_o a}{m} + F$$

the amount being absorbed being

(2) $(1 - m)\left[\frac{\pi b_o a}{m} + F\right]$

Neglecting the small amount of light flux lost through the lighting and observing windows, the sum of (1) and (2) must be equal to the total flux received, since it all escapes through the opening or is absorbed. Therefore, we have

(3) $\pi b_o c'' + c'mF + (1-m)\left[\frac{\pi b_o a}{m} + F\right] = F$

Solving (3) for b_o,

(4) $b_o = \frac{m^2 F(1-c')}{\pi[mc'' + a(1-m)]} = \frac{m^2 Fa}{\pi[mc'' + a(1-m)]}$

If the hole is covered with a flat surface having a reflection factor m_x, then $(1-m_x)$ times the flux incident on the test surface will be absorbed. In that case we find that

(5) $(1-m_x)(\pi b_x c'' + c'mF) + (1 - m)\left[\frac{\pi b_x a}{m} + F\right] = F$

Solving for b_x, we have

(6) $b_x = \frac{m^2 F(a + c'm_x)}{\pi[c''m(1-m_x) + a(1-m)]}$

If $m_x = m$, *i.e.*, if the test surface has the same reflection factor as the sphere surface, then (6) becomes

(7) $b = \frac{m^2 F(a + c'm)}{\pi[(1-m)(c''m + a)]}$

It is possible to measure the relative brightness of the sphere wall under the various conditions, and from these measurements m and m_x can be computed.

Let $\frac{b}{b_o} = K$ and $\frac{b_x}{b_o} = K_x$

Then

$$(8)\quad K=\frac{\dfrac{m^2F\,(a+c'm)}{\pi(1-m)\;(c''m+a)}}{\dfrac{m^2Fa}{\pi[mc''+a(1-m)]}}=\frac{(mc'+a)[mc''+a(1-m)]}{a(mc''+a)\;(1-m)}$$

Clearing of fractions and collecting,

$$(9)\quad m^2[c'(c''-a)+Kac'']+ma[c'+(c''-a)\;(1-K)]+a^2(1-K)=0.$$

Since c', c'' and a are dimensional constants of the sphere, and K can be measured, m, the reflection factor of the sphere surface, can be evaluated from this equation.

$$(10)\quad K_x=\frac{\dfrac{m^2F(a+c'm_x)}{\pi[c''m\,(1-m_x)+a(1-m)]}}{\dfrac{m^2Fa}{\pi[mc''+a(1-m)]}}=\frac{(a+c'm_x)[mc''+a(1-m)]}{a[mc''\,(1-m_x)+a(1-m)]}$$

$$(11)\quad m_x=\frac{a(K_x-1)\;[c''m+a(1-m)]}{ac'(1-m)+mc''(c'+K_xa)}$$

The above equations have been derived for the case where the direct light from the lamp is incident on the sphere wall. While it is possible to use the reflectometer with that arrangement, it is much more sensitive to changes of reflection factor of the test surface, and therefore more accurate, if it is arranged so that the direct light is incident on the test surface. The most convenient method is to use the former arrangement and determine the value of m from equation (9), then arrange the instrument so that the light is incident directly on the test surface, as shown in Fig. 1 *b*. By using for a reference standard a surface painted with the sphere paint, it is then possible to evaluate in absolute measure the reflection factor of any test surface. The theory of the instrument when so used is as follows:

Let c', c'', a, m, m_x and F have the same meaning as before.

b' = brightness of sphere surface when the hole is covered with a test surface of factor m_x.

$$R=\frac{\text{value of } b' \text{ when hole is covered with surface } m''}{\text{value of } b' \text{ when hole is covered with surface } m'}.$$

Since unit area of the sphere surface of brightness b' emits $\pi b'$ lumens, it must be receiving $\frac{\pi b'}{m}$ lumens. Hence the sphere area, a, absorbs $(1-m)\;\frac{\pi b'a}{m}$ lumens. The test surface receives $F+c''\pi b'$ and absorbs $(1-m_x)\;(F+c''\pi b')$ lumens. Hence, since

all the light is absorbed except that negligible amount which escapes through the lighting and observing holes, we have

$$(13)\quad (1-m)\frac{\pi b' a}{m} + (1-m_x)\,(F + c''\pi b') = F$$

Solving for b',

$$(14)\quad b' = \frac{mm_x F}{\pi[a(1-m) + c''m(1-m_x)]}$$

$$(15)\quad R = \frac{\dfrac{mm'' F}{\pi[a(1-m) + c''m(1-m'')}}{\dfrac{mm' F}{\pi[a(1-m) + c''m(1-m')]}} = \frac{m''[a(1-m) + c''m(1-m')]}{m'[a(1-m) + c''m(1-m'')]}$$

In the special case where $m' = m$, we have

$$(16)\quad m'' = m_x = \frac{Rm[a(1-m) + mc'']}{(1-m)\,(a + c''m) + Rm^2c''}$$

V. EXPERIMENTAL RESULTS.

In order to verify the theory of this instrument a graded series of test objects was made up. Neutral gray objects were obtained by mixing black drawing ink and lampblack with a white cement (Keene's Fine). They were surfaced with coarse sandpaper, resulting in fairly good diffusers. Those having reflection factors below 50 per cent. were better diffusers than those above that value. This appears to be characteristic of this material, possibly of others, and will be treated more fully in a future paper.

The test objects were next tested for reflection factors by means of the apparatus shown in Fig. 2. The objects were illuminated normally by a 60-watt vacuum tungsten lamp, and the surface brightness was measured by point-by-point observations. The amount of flux reflected was then calculated by applying the proper factors, and since the incident flux was known, the ratio of the reflected to the incident flux gave the reflection factors of the test surfaces. The greatest difficulty in that method of measurement is the standardization of the apparatus to read surface brightness, as it involves very considerable practical difficulties, chief among which is the great intensity step which must be taken. Much work was done on this part of the problem, and it is believed that the uncertainty in that standardization was not very large.

A number of the cement standards and a block of magnesium carbonate were measured by both methods of using the instru-

ment as described above. By the use of equation (9) the reflection factor of the sphere surface was found to be 88.3 per cent. This value was then substituted in the equations and the reflection factors of the various test objects were computed. The results of the measurements in the three different ways are shown in Table I.

TABLE I.

Reflection Factors by Three Methods of Measurement.

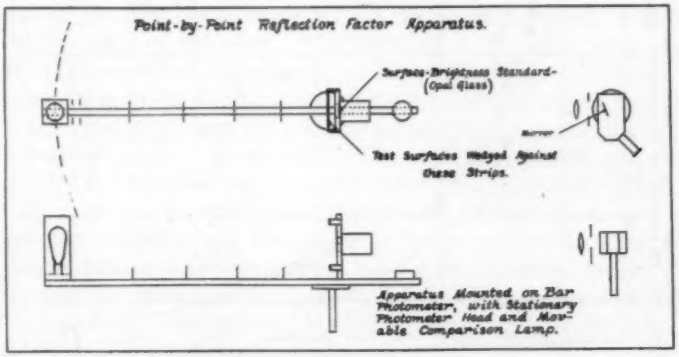
Test object	Point-by-point measurement	Sphere reflectometer	
		Direct light on test objects	Direct light on sphere wall
	per cent.	*per cent.*	*per cent.*
Cement standard No. 1.....	17.7	18.4	21.9
Cement standard, No. 2.....	23.7	23.9	28.2
Cement standard, No. 3.....	31.5	31.6	35.2
Cement standard, No. 4.....	36.3	37.1	39.6
Cement standard, No. 5.....	42.9	42.9	46.4
Cement standard, No. 6.....	60.0	60.7	62.8
Cement standard, No. 7.....	67.4	67.3	70.2
Cement standard, No. 8.....	81.9	81.4	82.7
Cement standard, No. 9.....	86.5	85.5	86.8
Cement standard, No. 10.....	90.8	90.4	89.6
Magnesium carbonate.....	99.3	99.1	98.7

It will be noted that the agreement betwen columns 2 and 3 is almost perfect, and that for reflection factors above 80 per cent. the fourth column is in satisfactory agreement. For factors below 80 per cent. the method represented by the fourth column is rather insensitive, the error in the result being larger than the error of the photometer reading with the actual conditions existing in the experimental instrument. This is clearly shown by the curves in Fig. 3. When the direct light is incident on the test surface, the error in the factor is always less than the error of the photometer reading. Hence, it may safely be stated that at least a part of the discrepancy between columns 2 and 4 may be attributed to experimental error. The fact that the differences are all in the same direction for the objects having low reflection factors might lead to the belief that not all the discrepancy could be explained this way, and that it might be necessary to look farther for an additional reason. It is possible that the reflection factor may be higher for diffused light than for light having a large normal component. If this is true, the differences are in the direction to be expected. The sensitivity of the instru-

ment when used with diffused illumination of the test surfaces would be greatly increased if the reflectometer were painted with a paint having a higher reflecting power and it might then be possible to determine whether this is a real departure from the theory as developed. The author has made up paints having a factor of about 94 per cent., which would be much better for this application.

It will be noted that the measurements by the three methods give about 99 per cent. for magnesium carbonate, whereas the previously accepted value was only 88 per cent. It did not seem

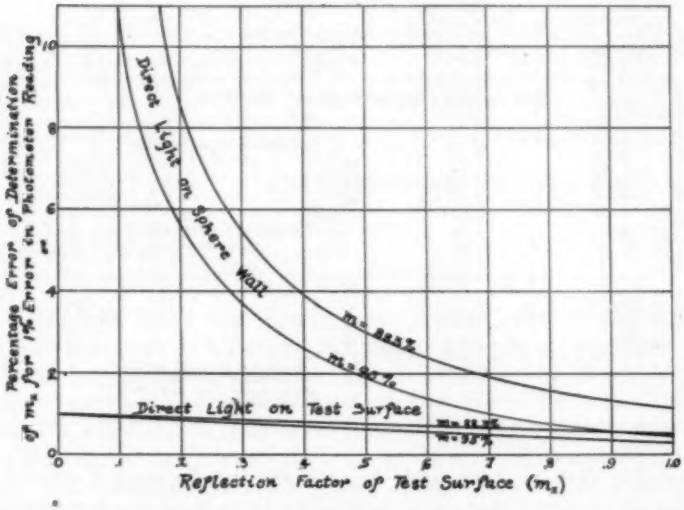
FIG. 2.



possible that its value could be so high, and the author was reluctant to accept that value as reliable, in spite of the agreement of the three methods, unless verified by still another method. Hence, steps were taken to verify it by a fourth absolute method, which was described above as a modification of the sphere method used by Mr. Little. For this purpose a small disk of magnesium carbonate was made up from the block previously tested. It was placed in a sphere and a very narrow beam of light was projected through a small hole in the sphere wall onto the sphere surface at a point unscreened from the observation window, then onto the magnesium carbonate disk so placed that none of the first reflected light from it could reach the observation window. The ratio of the brightness of the window in the second case to that in the

first case is the reflection factor of the test surface. This measurement gave a factor of 98.9 per cent., which, on the basis of the consistency of the photometric readings, appeared to be thoroughly reliable. Hence, it appears that the reflection factor of this particular block of magnesium carbonate is 99 per cent. Another block, obtained about three weeks later from the same source, has a factor of approximately 98 per cent., while another block which has been in the laboratory for about two or three

FIG. 3.


Percentage Error of Determination of m_s for 1% Error in Photometer Reading
Direct Light on Sphere Wall
Direct Light on Test Surface
Reflection Factor of Test Surface (m_s)

years is appreciably darker than either of these. The author has not investigated the reproducibility of such surfaces for precision standards.

It should be pointed out that magnesium carbonate is not a perfect diffuser, hence this must be taken into account if it is desired to use it as a standard of surface brightness. The block which was tested by point-by-point measurements was surfaced by scraping with a sharp glass edge. When illuminated normally its brightness at about 50° from the normal to the surface was the same as that of a theoretically perfect diffuser emitting the same total flux.

VI. EFFECT OF SPECULAR REFLECTION FROM TEST SURFACES.

As mentioned above, the surfaces with which these tests were carried out were fairly good diffusers. Some other surfaces which have been examined by point-by-point measurements have been ordinary semi-matt surfaces. An examination of these data shows that when the light is incident normally the angle at which the surface has the same brightness as a perfect diffuser emitting the same total flux is usually about 45° to 50° from the normal, which is also approximately the angle between the normal and the line joining the observation window and the centre of the test surface in the instrument used in these tests. The difference between the surface brightness of the test surface at 50° and a perfect diffuser emitting the same total flux does not exceed 3 to 5 per cent. in the case of all the surfaces tested, and is usually below 3 per cent. The effect of a deviation of 5 per cent. from perfect diffusion at 50° for a surface of reflection factor 80 per cent. has been calculated for a reflectometer having 10 per cent. of its area cut off, and painted with a paint having a reflection factor of 90 per cent. The calculation shows that the error of the determination would be less than .5 per cent. Hence, it appears safe to state that the error of determination due to specular reflection will not exceed 1 or 2 per cent in a reflectometer having these dimensions, unless an excessive amount of the specular reflection of direct light is incident on the observation window. This will not hold true unless the specular reflection takes place only at the first surface.

If it is desired to determine the reflection factor of a mirror, silvered on the back, this may be done by first directing a narrow beam of light into the opening of the reflectometer, then letting the same beam be reflected from the mirror and be directed into the reflectometer when the mirror is placed at an appreciable distance from the reflectometer. The ratio of the brightness of the observation window in the second case to that in the first will be the reflection factor of the mirror.

VII. PRECAUTIONS IN USE OF REFLECTOMETER.

In the use of the reflectometer as described above, certain precautions are necessary. Some of them are as follows:

(*a*) The dimensions should be precisely determined in order to fix the values of the constants c', c'' and a.

(*b*) In painting the sphere and the flat surface by means of which its reflection factor is determined, care must be taken to make the flat surface as nearly as possible the same in reflecting power as the sphere surface.

(*c*) When calculating the reflection factor of the sphere surface by the use of equation (9), the figures should be carried out as far as possible, as the equation involves differences of numbers which are nearly equal, and a small error in calculation may make an appreciable error in the result. The use of logarithms for this step is recommended, though slide rule calculations are sufficiently accurate in working up test data for observations taken with direct light on the test surfaces.

(*d*) When the direct light is incident on the sphere wall, care must be taken to prevent any direct light from escaping through the large opening, *c''*.

The most symmetrical arrangement of the reflectometer is obtained if the observation hole and the side lighting hole are about 90° from the centre of the portion cut off, and 90° from each other. This is the arrangement shown in Fig. 1 *a*.

VIII. CONCLUSION.

In conclusion the most important points brought out by this paper may be summarized as follows:

1. Five "absolute" methods of measuring reflection factors are described, at least three of which are apparently new. Measurements on magnesium carbonate by four of these methods give values which are in excellent agreement.

2. The numerical value of the reflection factor for magnesium carbonate which has been in use for many years, *viz.*, 88 per cent., is in error. This fact was first publicly pointed out by the author at Chicago in October, 1919. The actual value of its diffuse reflection factor is approximately 99 per cent., but the degree of reproducibility of this value with materials from different sources is unknown.

3. The method described above for the use of an incomplete sphere as a reflectometer furnishes two new absolute methods for the determination of diffuse reflection factors. The determination of the reflection factor of the sphere surface is only an incidental step in the use of the instrument, and is not its principal object.

4. The instrument just described should find a large field of usefulness in photometry and illuminating engineering, and furnishes a method of measuring the reflection factors of surfaces *in situ*. Apparently no other instrument which is accurate and portable has been proposed for this purpose. It can be adapted for use with any good type of portable photometer.

5. It should be strongly emphasized that the reflection factor of any *colored* surface is dependent on the color of the incident light and that measurements by this or any other method will give its reflection factor only under the particular conditions of the test. Hence, under such conditions the principal value of the measurement is to indicate the approximate value of the factor, but this is all that is usually required.

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[1] "A New Method and Instrument for Determining the Reflecting Power of Opaque Bodies," P. G. Nutting, *Trans. Illuminating Engineering Society*, Vol. 7, p. 412, 1912.

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[4] "Measurement of Reflection Factor," M. Luckiesh, *Elec. World*, Vol. 69, p. 958, 1917.

[5] "Photometric Units and Nomenclature," E. B. Rosa, *Bulletin of the Bureau of Standards*, Vol. 6, p. 543, 1909–10; also Scientific Paper No. 141.

BUREAU OF STANDARDS.

A NEW MICROPHOTOMETER FOR PHOTOGRAPHIC DENSITIES.

BY

W. F. MEGGERS and PAUL D. FOOTE.

I. INTRODUCTION.

PROBLEMS involving the necessity of measuring the photographic density of a minute portion of a photographic plate have called into existence several types of microphotometers, such as the well-known Hartman [1] instrument, the Koch [2] registering microphotometer and the new microphotometer recently described by Fabry and Buisson.[3]

In the course of some recent experiments on low voltage arc spectra of cæsium, the writers required the use of a microphotometer to measure photographic densities of spectral lines. None of the above-mentioned instruments being at hand, the instrument which was employed by the writers over three years ago in some photographic density work by the addition of a few slight changes, proved so satisfactory, accurate, and efficient a microphotometer as to merit serious consideration as a piece of laboratory apparatus. The instrument (Fig. 1) is essentially the micropyrometer described by Burgess,[4] except that a microscope of higher power was employed. The photographic plate was mounted just below the objective of the microscope on a horizontal bed, movable with a graduated screw, and was illuminated beneath by an intense beam of light from a tungsten ribbon gas-filled lamp. Light transmitted by a small portion of the photographic plate, the image of which was adjacent to that of the tip of the photometer lamp, was matched with equal filament brightness by adjusting the current through the lamp. The ammeter readings are readily translated into measurements of photographic density.

II. CALIBRATION.

To define the state of blackness of a developed photographic plate, or in general of any absorbing layer, a beam of light is

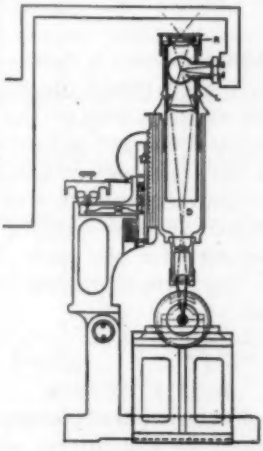
[1] *Zeitschr. f. Inst.* **19**, p. 97, 1899.
[2] *Ann. de Phys.* (4) **39**, p. 705, 1912.
[3] *J. de Phys.* (5) **9**, p. 37, 1919.
[4] *Bull. Bureau of Standards*, **9**, p. 475, 1913.

passed through it and the intensity I of the incident beam is compared with the intensity T of the transmitted beam. The ratio $O = I/T$ is called the opacity, and the value of this ratio is 1 for a perfectly transparent plate. It is more convenient, however, to use the logarithm (taken to the base 10) of this quantity and call it density, D. Then:

$$D = \log O = \log I/T$$

so that a perfectly transparent plate has zero density and a density of 1 indicates that 1/10 of the incident light is transmitted by a

FIG. 1.



Microphotometer.

plate. In general, slightly different values of density are found from measurements with different-colored lights. It is, therefore, advantageous to make density measurements with homogeneous light rather than with white light, since the phenomena then obey the above simple law and the difficulties inherent in heterochromatic photometry are evaded.

These advantages are possessed by the microphotometer in which the usual "monochromatic" red-glass screen used in optical pyrometers is employed. Of course, a suitable monochromatic screen of any other color could be used in the same way. The

densities measured with red light will be the same as for any other color if the photographic plate is non-selective.

There are several methods of calibration and use of this type of photometer, of which the following appeared the most satisfactory: The instrument with a red glass ocular was calibrated as a pyrometer by sighting upon a small black body, the temperature of which was measured by an optical pyrometer, thus obtaining the relation between the temperature of the source and the current through the photometer lamp sufficient for a match. If the red glass screen is sufficiently monochromatic, we may neglect the variation of the "effective wave-length" of transmission with the temperature of the source.[5] The particular glass employed had an effective wave-length of approximately $\lambda = 0.65\mu$.

In use the illuminating source is maintained at any constant brightness sufficiently high to permit observation on lines of the highest density. The measurements of the current required to match against any spectral line and against the clear film in the neighborhood of this line are made and are converted into "temperatures" by use of the calibration plot of the instrument. If θ_1 is the absolute temperature observed for the line and θ_2 the corresponding temperature for the clear film (fog) the true density D of the line, fog correction being considered, follows at once from Wien's law:

$$D = \frac{c_2 \log e}{\lambda}\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right) = 9590\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right)$$

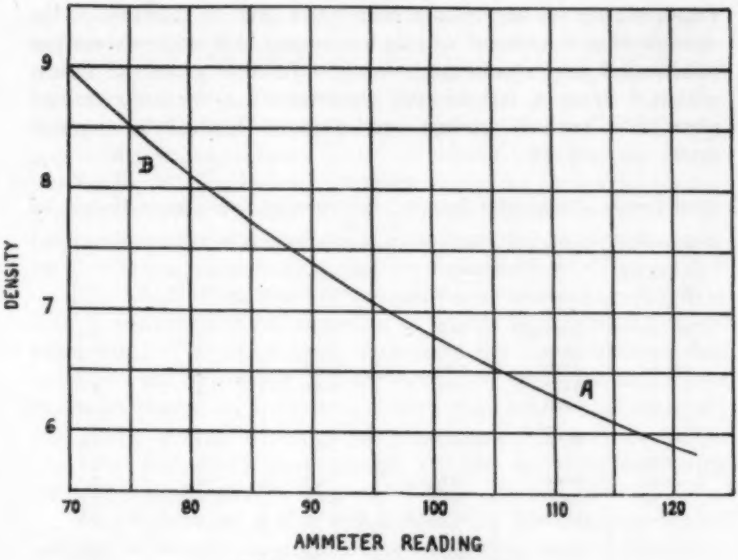
For convenience of use, instead of plotting current through the lamp *versus* temperature, we plot current *versus* $9590/\theta$, obtaining for the particular instrument employed, the curve shown in Fig. 2. Thus, suppose the current i_a is observed for any fog reading on clear film and i_b when sighted upon the line. The true density of the line is given by the difference in ordinates of the points A and B.

Another method of calibration, in some respects more satisfactory, is to sight upon the illuminating source and vary the intensity by means of sectored disks. Still another method is to use screens which have been measured by other photometers. Screens calibrated for white light may be employed when plates

[5] *Bull. Bur. of Stand.* 12, p. 483, 1915.

similarly treated are measured, thus eliminating the slight difference in density arising from measurements in the usually employed white light and red light on account of any slight selectivity of the gelatine. However, the effect of scattering of light by the photographic film is many times greater than that of the selec-

FIG. 2.



Calibration curve of microphotometer.

tive absorption and requires special attention to the type of illumination employed, as discussed later.

III. APPLICATION.

The primary object for which this microphotometer was assembled and calibrated was to measure the photographic densities of narrow spectral lines. To accomplish this, the photographic plate was mounted in the object plane of the microscope and moved horizontally to bring the images of different spectral lines or parts of the plate in coincidence with the image of the tip of the pyrometer lamp filament. This shift was effected by turning a screw which was provided with a revolution counter,

and a head divided so that 1 division corresponds to 5μ in linear motion of the bed on which the photographic plate is mounted. An important feature of this arrangement is that the tip of the pyrometer lamp filament may be regarded as a filar in the eye-piece and the relative positions or wave-lengths of spectral lines on a plate are measured at the same time that their photographic densities are measured. The measurement of densities has been explained above, and it only remains to give an example of the simultaneous double use of this instrument as a photo-measuring micrometer and a microphotometer. Table I gives the results obtained from a one-minute exposure of a dicyanin stained plate to a six-volt cæsium arc spectrum formed by a small prism spectograph.

TABLE I.

Wave Lengths, Photographic Densities, and Estimated Intensities in Cæsium Arc Spectrum.

Micrometer readings	Wave length		Photographic densities	Estimated intensities	I/D
	Computed	Accepted			
178.81	4555.30*	4555.30	0.78	5	6.4
179.47	4592	4593	.73	5	6.9
181.64	5656	5664	.58	4	6.9
2.92	5837	5845	1.01	7	7.0
4.01	6006	6010	1.08	8	7.4
4.15	6029	6034	.40	3	7.5
5.19	6211	6213	1.16	9	7.8
5.93	6353	6355	.93	7	7.5
7.04	6586	6587	1.04	8	7.7
7.64	6723.18*	6723.18	1.31	9	6.9
8.05	6825	6827	.59	4	6.8
8.26	6878	6873	.93	7	7.5
8.67	6975	6973	1.38	10	7.2
9.53	7229	7229	1.03	8	7.8
199.72	7286	7280	.93	7	7.5
200.72	7615	7610	.76	5	6.6
1.61	7945	7944	.45	3	6.7
1.80	8024	8017	.26	2	7.7
1.95	8086	8080	.17	1	6.0
2.92	8521.12*	8521.12	1.16	9	7.7
203.73	8946	8943	.28	2	7.2

The first three columns of Table I illustrate the use of the microphotometer for wave-length measurements at the same time that photographic densities (column 4) of the spectral lines are measured. Three wave-lengths marked with asterisks were used in wave-length computations by the Hartmann Formula.

In column 5 the visually estimated intensities of the spectral lines are given. These estimates were made according to the usual practice in spectroscopy of assigning values from 1 to 10 to the intensities of spectral lines recorded on a photographic plate. According to Fechner's law, that "the intensity of sensation increases as the logarithm of the stimulus," the natural tendency in formulating a scale of opacities is to choose a geometric series so that the ratio of the transmissions of any two adjacent steps is constant throughout the series. In other words, the apparent contrasts between successive steps are equal only if the logarithms of the transmissions differ by equal amounts. Visually estimated intensities of spectral lines appear to obey this law very approximately, as is shown in the last column of Table I, in which the ratios of visually estimated intensities to measured photographic densities is given. If the estimated intensities are divided by 7, in this particular example, the quotients will be in close agreement with the photographic densities or logarithms of opacities.[6]

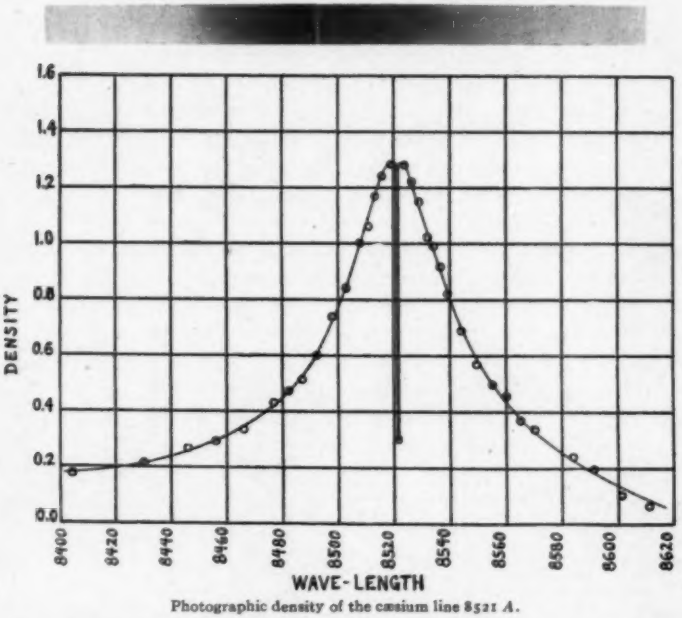
Fig. 3 is an illustration of the possible use of the microphotometer in measuring the distribution of energy in a very wide spectrum line. The photograph represents the appearance of the infra-red line 8521.12 *A* in the 120-volt arc spectrum of cæsium as photographed on a dicyanin stained plate with a concave grating giving a scale of 10 *A* per millimetre. The curve below was obtained from photographic density measurements with the microphotometer.

This instrument is also well adapted for the measurement of spectral sensitivity of various types of photographic plates after being exposed in a spectograph to a continuous spectrum. A typical example of this is shown in Fig. 4, which represents microphotometer measurements of density on a Seed 23 plate stained with dicyanin and developed after an exposure of one minute in

[6] It is readily possible, often advisable and sometimes the practise of spectroscopists, to use more than 10 degrees of opacity in estimating intensities of spectral lines. The average spectrogram perhaps does not give a density of more than 2 for the strongest line on the plate, and this suggests that the upper limit of the scale of estimated intensities might conveniently be extended to 20, or even to 200. Then a larger number of steps would be available in a finer discrimination of opacities, and if the estimates follow Fechner's law, the factor 10 or 100 will readily permit a rough, but often sufficiently precise, transformation from visually estimated intensities to photographic densities.

a prism spectograph which was directed at a black body furnace at 1450° C. The sensitivity of this stained plate to red light appears to be enormously greater than its blue sensitiveness, but the curve should really be corrected to equal energy distribution in the spectrum. To accomplish this, "characteristic curves" for different wave-lengths were drawn from microphotometer measure-

FIG. 3.



Photographic density of the cæsium line 8521 *A*.

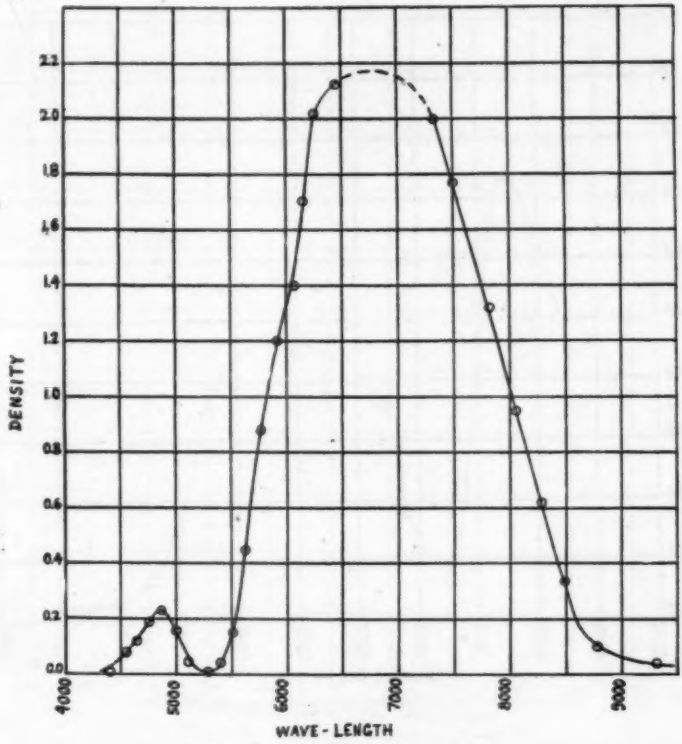
ments on exposures of different durations and then the plate sensitivity was reduced to the relative densities which would be expected if the spectrum possessed equal energy for all wave-lengths. This is shown in Fig. 5.

As a test of the accuracy of density measurements obtained with the microphotometer, some strips of photographic plates prepared in a sensitometer for the purpose of obtaining the *H* and *D* characteristic curves[7] were very carefully measured with a

[7] Hurter and Driffield: *Jour. Soc. Chem. Ind.*, May, 1890, p. 455.

Koenig-Martens polarization photometer and then with the new microphotometer. The densities measured with the microphotometer were found to be considerably larger than those given by the polarization instrument and the chief reason for this was

FIG. 4.

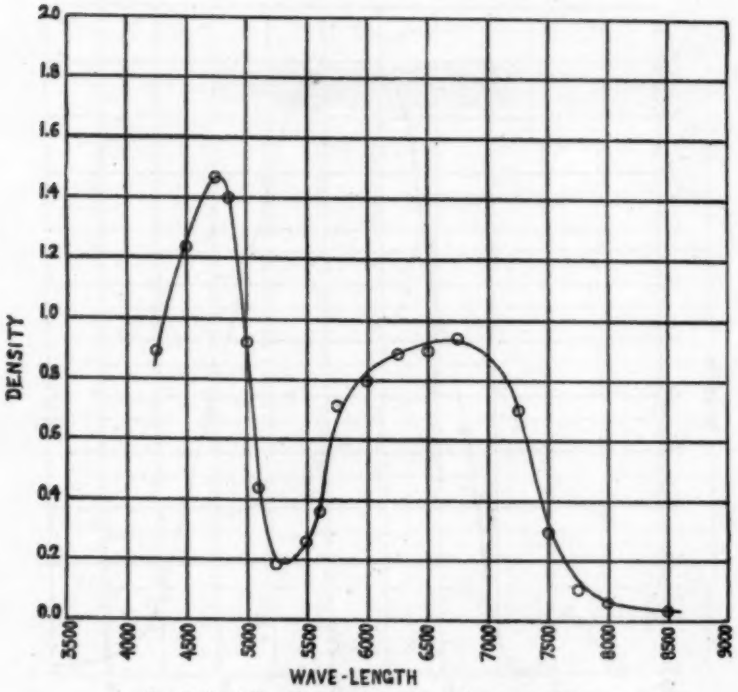


Sensitivity of a Seed 23 plate stained with dicyanin.

traced to the manner in which the photographic plate was illuminated in the two instruments. In the polarization photometer the photographic strip is lighted by lamps in a box of opal glass so that very diffuse light illuminates the plate. This diffuse light is scattered and partly reradiated by developed silver particles in

the photographic plate, and gives the plate apparently a higher transmission and lower density than it really possesses. Practically the same low densities were obtained with the microphotometer when two layers of diffusing ground glass were placed just

FIG. 5.

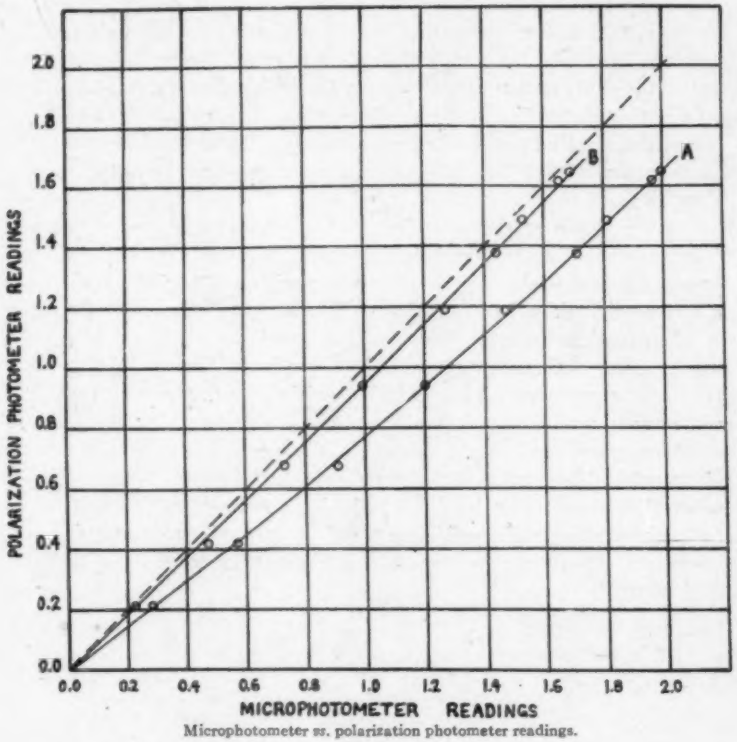


Sensitivity of dicyanin stained plate to equal energy spectrum.

behind the photographic plate. This is illustrated in Fig. 6, in which the densities of a sensitometer plate as measured with the polarization photometer are plotted as ordinates, and those measured with the pyrometer microphotometer as abscissæ. If both instruments gave the same densities the values would lie on a dashed line at 45°. Curve *A* shows densities determined with the microphotometer when the photographic plate was illuminated

with parallel light and curve *B* when diffuse light from two layers of ground glass illuminated the plate. This experience calls attention to the importance of specifying the character of the illumination when measurements are made on the light transmission of diffusing media.[8]

FIG. 6.



Microphotometer *vs.* polarization photometer readings.

SUMMARY.

The new microphotometer for measuring photographic densities is essentially the micropyrometer described in the *Bulletin* of

[8] Absorption and diffusion of light in the developed photographic plate have been studied by Callier, who also presents some of the photographic consequences of the scattering of light. *Zeitschr. f. Wiss. Phot.* 7, p. 257, 1909.

the Bureau of Standards (Vol. ix, p. 475, 1913), except that a microscope of higher power is used. The photographic plate is mounted just below the objective of the microscope on a horizontal bed, movable with a graduated screw, and is illuminated beneath by an intense beam of light from a tungsten ribbon lamp. Light transmitted by a small portion of the photographic plate, the image of which is adjacent to that of the tip of the pyrometer lamp, is matched with equal filament brightness by adjusting the current through the lamp. The ammeter readings are readily translated into measurements of photographic density. If the tip of the pyrometer lamp filament be regarded as a filar in the eyepiece, the relative positions or wave-lengths of spectral lines on a plate are measured at the same time that their photographic densities are measured.

An example of the use of this microphotometer for measuring wave-lengths and densities of spectral lines is given, and it appears that the visually estimated intensities of such lines form a geometric series in accordance with Fechner's law.

Illustrations of this microphotometer's application to the measurement of energy distribution in a broad spectral line and to the measurement of spectral sensitivity of a photographic plate are given.

Measurements of photographic density when the plate is illuminated by parallel light are found to be much larger than when the illumination is diffuse. This experience calls attention to the importance of specifying the character of the illumination when measurements are made on light transmission of diffusing media.

BUREAU OF STANDARDS,
January 20, 1920.